

4/MTH-252 Syllabus-2023

2026

(May-June)

FYUP : 4th Semester Examination

MATHEMATICS

(Dynamics—I)

(MTH-252)

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions.*

Answer **four** questions, selecting **one** from each Unit.

UNIT—I

1. (a) A body moving in a straight line OAB with S.H.M. has zero velocity when at the points A and B whose distances from O are a and b respectively and has a velocity v when half-way between them. Show that the complete period

is $\frac{\pi(b-a)}{v}$.

6

- (b) A particle of mass m , is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle be released from rest, show that the distance fallen through in time t is

$$g \frac{m^2}{\mu^2} \left\{ e^{-\frac{\mu t}{m}} - 1 + \frac{\mu t}{m} \right\} \quad 6$$

- (c) A particle starts from rest at a distance a from the centre of force which attracts inversely as the distance. Prove that the time of arriving at the centre

$$\text{is } a \sqrt{\frac{\pi}{2\mu}}. \quad 6$$

2. (a) Show that the time of descent to the centre of force, the force varying inversely as the square of the distance from the centre, through the first half of its initial displacement is to that through the second half is $\pi + 2 : \pi - 2$. 6

- (b) A particle is projected in a medium whose resistance is proportional to the cube of the velocity and no other forces act on the particle. While the velocity diminishes from v_1 to v_2 and the particle traverse a distance d in time t , show that

$$\frac{d}{t} = \frac{2v_1v_2}{v_1 + v_2} \quad 5$$

- (c) A particle is attracted by a force to a fixed point varying inversely as (distance) ^{n} . If the velocity acquired in falling from rest at infinity to a distance a from the centre be equal to the velocity acquired in falling from rest at a distance a to a distance $\frac{a}{4}$, then show that $n = \frac{3}{2}$. 7

UNIT—II

3. (a) A particle moves on the curve $y = a \log \sec \left(\frac{x}{a} \right)$ in such a way that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature. 5

- (b) If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity. 3

- (c) Two bodies are projected from the same point in directions making angles α_1 and α_2 with the horizontal and strike at the same point in the horizontal plane through the point of projection.

If t_1, t_2 be their time of flights, show that

$$\frac{t_1^2 - t_2^2}{t_1^2 + t_2^2} = \frac{\sin(\alpha_1 - \alpha_2)}{\sin(\alpha_1 + \alpha_2)} \quad 6$$

(d) If the tangential and normal acceleration of a particle describing a plane curve be constant throughout, prove that—

(i) the radius of curvature at any time

t is given by $\rho = (at + b)^2$;

(iii) the angle ψ through which the direction of motion turns in time t is given by $\psi = A \log[1 + Bt]$; where A and B are constants. 5

4. (a) A point describes a circle of radius a with a uniform speed v . Show that the radial and transverse accelerations are $-\frac{v^2}{a} \cos \theta$ and $-\frac{v^2}{a} \sin \theta$, if a diameter is taken as initial line and one end of this diameter as pole. 6

(b) Particles are projected simultaneously in the same vertical plane from the same point. Show that the locus of the foci of all trajectories is a parabola, when for each trajectory there is the same (i) horizontal velocity, (ii) initial vertical velocity and (iii) time of flight. 6

(c) A ball is projected so as to just clear two walls, the first of height a at a distance b from the point of projection and the second of height b at a distance a from the point of projection. Show that the range on the horizontal plane is $\frac{a^2 + ab + b^2}{a + b}$ and the angle of projection exceeds $\tan^{-1} 3$. 7

UNIT—III

5. (a) Prove that for a system of conservative forces, the work done upon a particle of mass m by the forces is equal to the change in the kinetic energy of the mass. 6

(b) A shell of mass $(m_1 + m_2)$, moving with velocity u , breaks up into two masses m_1 and m_2 , which move in the same direction with relative velocity v . Show that the energy of explosion is given by

$$E = \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} v^2 \quad 6$$

(c) Three equal spheres are in a straight line on a table and one of these moves towards the other two which are at rest

and not in contact. If $e = \frac{1}{2}$, show that three impacts will take place and the ultimate speeds of the spheres will be in the ratio 13 : 15 : 36. 7

6. (a) A gun is mounted on a gun-carriage movable on a smooth horizontal plane and the gun is elevated at an angle α to the horizon. A shot is fired and leaves the gun in a direction inclined at an angle θ to the horizon. If the mass of the gun and the carriage be n times that of the shot, show that

$$\tan \theta = \left(1 + \frac{1}{n}\right) \tan \alpha \quad 7$$

- (b) If two equal smooth spheres, which are perfectly elastic, impinge at right angles, show that their directions after impact will also be at right angles. 6

- (c) The velocity of a ball A after its oblique impact on a ball B has the same magnitude and direction as the velocity of B before impact. Show that the ratio of the masses of the two balls is equal to the coefficient of restitution. 6

UNIT—IV

7. (a) A heavy particle of weight W attached to a fixed point by a light inextensible string, describes a circle in a vertical plane. The tensions in the string has values mW and nW respectively when the particle is at the highest and the lowest points of its path. Show that $n = m + 6$. 6

- (b) A particle moves under gravity in a vertical circle sliding down the convex side of the smooth circular arc. If its initial velocity is that due to a fall to the starting point from a height h above the centre, show that it will fly off the circle

when at a height $\frac{2}{3}h$ above the centre. 8

- (c) If a particle slides down from rest at the vertex of a cycloid, whose axis is vertical and vertex upwards, show that the velocity at any point is due to fall from the tangent at the vertex. 5

8. (a) Show that for a particle, sliding down the arc and starting from a cusp of a smooth cycloid whose axis is vertical and vertex lowest. Show that the velocity is maximum when it has described half the vertical height. 7

- (b) A particle is projected with velocity V from the cusp of a smooth cycloid, whose axis is vertical and vertex lowest, down the arc. Show that the time of reaching the vertex is

$$2\sqrt{\frac{a}{g}} \tan^{-1} \left(\frac{\sqrt{4ag}}{V} \right) \quad 7$$

- (c) A particle is projected along the inside of a smooth vertical circle of radius a from the lowest point. Show that the velocity of projection required in order that after leaving the circle the particle may pass through the centre is

$$\sqrt{\frac{1}{2}ag(\sqrt{3}+1)}.$$

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